The limits of θ are 0 and $\frac{1}{2}\pi$ and doubled; of v, $+r$ and $-r$; of x, $2z$ and 0; of y, 0 and x and doubled; of w, v and r and doubled; of u, $-r$ and v and doubled. Area $PRQS=\frac{1}{2}y(w-u)$.

The whole number of ways four points can be taken in the circle is π^4r^8.

$$\therefore \triangle=\frac{16}{\pi^4r^8}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}\int_v^{r}\int_{-r}^{v}\tfrac{1}{2}y(w-u)d\theta dvdxydy2tdw2sdu$$

$$=\frac{8}{3\pi^4r^8}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}\int_v^{r}[4(a^2-v^2)^{\frac{3}{2}}+6vw\sqrt{(a^2-v^2)}+6a^2w\sin^{-1}(v/a)$$

$$+3\pi a^2w]ty^2d\theta dvdxdydw$$

$$=\frac{16}{3\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}\int_0^{x}(a^2-v^2)^{\frac{3}{2}}y^2d\theta dvdxdy$$

$$=\frac{16}{9\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}\int_0^{2z}(a^2-v^2)^{\frac{3}{2}}x^3d\theta dvdx$$

$$=\frac{64}{9\pi^3r^6}\int_0^{\frac{1}{2}\pi}\int_{-r}^{r}(a^2-v^2)^{\frac{7}{2}}d\theta dv=\frac{35r^2}{18\pi^2}\int_0^{\frac{1}{2}\pi}d\theta=\frac{35r^2}{36\pi}.$$

MISCELLANEOUS.

74. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

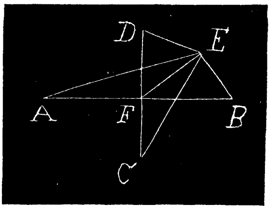
The longest diameter of a horizontal ellipse is $CB=2a=6$ feet. Its shortest diameter is $EF=2b=4$ feet, their intersection being at D. Find in an indefinite vertical plane passing through CB, a point $A=5$ feet$=c$ from D, the ellipse being seen from A as a circle.

III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

To find a point E in the vertical plane AEB at which DC, AB subtend the same angle, we proceed as follows: Let $DF=FC=b=2$, $AF=FB=a=3$, $FE=c=5$, $AE=x$, $BE=y$, $DE=CE=z$, $\angle DEC=\angle AEB=\theta$.

Then $c=\frac{1}{2}\sqrt{(2x^2+2y^2-4a^2)}=\frac{1}{2}\sqrt{(x^2+y^2+2xy\cos\theta)}$.

$\therefore 2(a^2+c^2)=x^2+y^2$; $4c^2=x^2+y^2+2xy\cos\theta$.



$\therefore x^2+y^2=2a^2+2c^2 \ldots\ldots (1)$.

$2xy\cos\theta=2c^2-2a^2 \ldots\ldots (2)$.

But $c=\frac{1}{2}\sqrt{(4z^2-4b^2)}=\frac{1}{2}\sqrt{(2z^2+2z^2\cos\theta)}$.

$\therefore z=\sqrt{(b^2+c^2)}$ and $\cos\theta=\dfrac{c^2-b^2}{c^2+b^2}$.

$\therefore 2xy=\dfrac{2(c^2-a^2)(c^2+b^2)}{c^2-b^2}$. $\therefore x+y=\dfrac{2\sqrt{(c^4-a^2b^2)}}{\sqrt{(c^2-b^2)}}$, $x-y=\dfrac{2c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}}$.

$$\therefore\ x=\frac{\sqrt{(c^4-a^2b^2)}+c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}},\ y=\frac{\sqrt{(c^4-a^2b^2)}-c\sqrt{(a^2-b^2)}}{\sqrt{(c^2-b^2)}}.$$

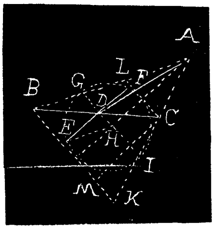
Substituting numbers we get $\cos\theta=\frac{21}{29}$, $\theta=43^\circ\ 36'\ 9''$.

$$x=\frac{\sqrt{(589)}+5\sqrt{(5)}}{\sqrt{(21)}}=7.73575 \text{ feet},\ y=\frac{\sqrt{(589)}-5\sqrt{(5)}}{\sqrt{(21)}}=2.85625 \text{ feet}.$$

$z=\sqrt{(29)}=5.38516$ feet.

IV. **Solution by A. H. BELL, Hillsboro, Ill.**

Let the vertical plane be ABK, A the vertex and BK the base of a right cone. The horizontal cutting plane BC is the major axis of the ellipse with D the projection of the minor axis, the cutting plane GI passing through D, and parallel to the base BK, is a circle and contains the minor axis of the ellipse. Revolving the circle 90° with the diameter GI as an axis, the chord EF is the minor axis of the ellipse; and s, s' are the foci. LC is a circle and parallel to the base BK of the cone.



$BD=DC=a=3$ feet, $ED=DF=b=2$ feet, $AD=c=5$ feet.

The properties of an ellipse give $s,\ s'=BL=CK$......(1).

$$BK\times CL=EF^2=4b^2\ \ldots\ldots\ (2).$$

$BC^2=BL^2+BK\times CL$. $\therefore\ BL=2(a^2-b^2)^{\frac{1}{2}}=4.4721360$......(3).

In the right triangle ADF, $AF=AI=AG=(b^2+c^2)^{\frac{1}{2}}=\sqrt{(29)}=5.3851648$.

$BG=GL=CI=\frac{1}{2}BL=2.2360680$.

$AC=AI-CI$. $AB=AI+CI=7.6212328$, $AC=3.1490968$, and the point A is determined.

NOTE. Radius, $GH=(\frac{29}{6})^{\frac{1}{2}}=2.198484326+=\left(\frac{b^2(b^2+c^2)}{2b^2+c^2-a^2}\right)^{\frac{1}{2}}$.

$DH=(GH^2-b^2)^{\frac{1}{2}}=(\frac{5}{6})^{\frac{1}{2}}=0.9128709$.

75. Proposed by J. C. NAGLE, A. M., M. C. E., Professor of Civil Engineering, State Agricultural and Mechanical College, College Station, Texas.

The water tank at the Nacogdoches River on the H. E. & W. T. Ry. is filled by a 3-inch pipe from a reservoir in which the water level is 6 feet above water in tank when full. The top diameter of tank is 17 feet, the bottom diameter is 19 feet, 8 inches, and the pipe projects 10 inches through the bottom. The depth is 13 feet, 6 inches. Find the time required to fill tank, taking the pipe as clean and free from sharp bends, except the right-angled one directly under tank. This bend is 12 feet below outlet of pipe, so that the total length of pipe is 1972 feet. Compare the result with the time of filling if the inlet pipe projected over top of tank instead of entering at the bottom.